J.H. Rindel, Computer simulation techniques for the acoustical design of rooms - how to treat reflections in sound field simulation. ASVA 97, Tokyo, 2-4 April 1997. Proceedings p. 201-208.

COMPUTER SIMULATION TECHNIQUES FOR ACOUSTICAL DESIGN OF ROOMS - HOW TO TREAT REFLECTIONS IN SOUND FIELD SIMULATION

Jens Holger Rindel
Institute of Acoustic Technology, Building 352,
Technical University of Denmark, DK-2800 Lyngby, Denmark
E-mail: jhr@dat.dtu.dk

ABSTRACT

The paper presents a number of problems related to sound reflections, and possible solutions or approximations that can be used in computer models. The problems include:

- specular and diffuse reflections,
- early and late reflections,
- scattering due to rough structure of surfaces,
- diffraction due to finite size of surfaces,
- curved surfaces, convex and concave surfaces,
- angle dependent absorption.

Due to the wave nature of sound it is very important for the quality of a simulation model, that diffusion and diffraction effects are included in a proper way. This can lead to more reliable computer models and at the same time, good results can be obtained with a minimum of calculation time.

INTRODUCTION

For the simulation of sound in large rooms there are two classical geometrical methods, namely the Ray Tracing Method and the Image Source Method. For both methods it is a problem that the wavelength or the frequency of the sound is not inherent in the model. This means that the geometrical models tend to create high order reflections which are much more accurate than would be possible with a real sound wave. So, the pure geometrical models should be limited to relatively low order reflections and some kind of statistical approach should be introduced in order to model higher order reflections. One way of introducing the wave nature of sound into geometrical models is by assigning a scattering coefficient to each surface. In this way the reflection from a surface can be modified from a pure specular behaviour into a more or less diffuse behaviour, which has proven to be essential for the development of computer models that can create reliable results.

SPECULAR AND DIFFUSE REFLECTIONS

Snell's Law of Reflection.

Specular reflections are used both in the Ray Tracing Method and in the Image Source Method. The Ray Tracing Method uses a large number of particles, which are emitted in various directions from a source point. The particles are traced around the room, and when a particle hits a surface it is reflected, which means that a new direction of propagation is determined, normally as a specular reflection. According to Snell's law, the angle of reflection is equal to the angle of incidence as known from geometrical optics.

The Image Source Method is based on the principle, that a specular reflection can be constructed geometrically by mirroring the source in the plane of the reflecting surface. In a rectangular box shaped room it is very simple to construct all image sources up to a certain order of reflection, and from this it can be deduced that if the volume of the room is *V*, the approximate number of image sources within a radius of *ct* is

$$N_{refl} = \frac{4\pi c^3}{3V} t^3 \tag{1}$$

This is an estimate of the number of reflections that will arrive at a receiver up to the time *t* after sound emission, and statistically this equation holds for any room geometry. In a typical auditorium there is often a higher density of early reflections, but this will be compensated by fewer late reflections, so on average the number of reflections increases with time in the third power according to the above equation.

Lambert's Law of Reflection.

The scattering of sound from surfaces can be quantified by a scattering coefficient, which may be defined as follows: The scattering coefficient δ of a surface is the ratio between reflected sound power in non-specular directions and the total reflected sound power. The definition applies for a certain angle of incidence, and the reflected power is supposed to be either specularly reflected or scattered. One weakness of the definition is, that it does not say how the directional distribution of the scattered power is; even if δ = 1 the directional distribution could be very uneven.

According to the above definition the scattered power P_{scat} can be expressed as:

$$P_{scat} = \delta P_{refl} = \delta(1-\alpha) P_{inc} \tag{2}$$

where P_{refl} is the total reflected power, P_{inc} is the incident power and α is the absorption coefficient of the surface. The scattering coefficient may take values between 0 and 1, where δ = 0 means purely specular reflection and δ = 1 means, that all reflected power is scattered according to some kind of 'ideal' diffusivity.

In computer models ideal diffuse reflections are normally assumed to follow Lambert's cosine law: In any direction (θ,φ) the intensity of scattered sound is proportional to cos θ, i. e. proportional to the projection of the wall area. The incident power on a surface exposed by a diffuse sound field would also obey Lambert's law, so this can be taken as one argument that this can be considered the ideal angular distribution for a diffuse reflection. For further discussion see ref. [1].

EARLY AND LATE REFLECTIONS

Image Source Method.

Early reflections should be determined with high accuracy in terms of level, arrival time and direction. This means that the Image Source Method is preferred, but the disadvantage of this method is very long calculation time for higher order reflections in rooms geometries, which are not simple rectangular box shapes. For that reason many computer models are hybrid models, which use ray tracing to find possible reflection sequences from a source in a room. From such a list of surface numbers it is possible to calculate the coordinates of the corresponding image source. The possible image sources thus generated are then tested as to whether they give a contribution at the chosen receiver position. This is called a visibility test and it can be performed as a tracing back from the receiver towards the image source.

It is, of course, common for more than one ray to follow the same sequence of surfaces, and discover the same potentially valid images. It is necessary to ensure that each valid image is only accepted once, otherwise duplicate reflections would appear in the reflectogram and cause errors. Therefore it is necessary to keep track of the early reflection images found, by building an 'image tree'.

Secondary Source Method.

There are good reasons to chose different methods for early and late reflections: The rapidly increasing density of reflections with time, eqn. (1), and a decreasing relevance of the details of each reflection as the theoretical models tends to deviate more and more from the physical truth with increasing reflection order. Some other method has to be used to generate a reverberation tail. This part of the task is the focus of much effort, and numerous approaches have been suggested, usually based on statistical properties of the room's geometry and absorption. One method, which has proven to be efficient, is the 'secondary source' method used in the ODEON program [2]. This method is outlined in the following.

The method is based on the same principle of ray tracing as used for early reflections, but after the transition from early to late reflections, the rays are treated as transporters of energy rather than explorers of the geometry. Each time a ray hits a surface, a secondary source is generated at the collision point. The energy of the secondary source is the total energy of the primary source divided by the number of rays and multiplied by the reflection coefficients of the surfaces involved in the ray's history up to that point. Each secondary source is considered to radiate into a hemisphere as an elemental area radiator. Thus the intensity is proportional to the cosine of the angle between the surface normal and the vector from the secondary source to the receiver. The intensity of the reflection at the receiver also falls according to the inverse square law, with the secondary source position as the origin. The time of arrival of a reflection is determined by the sum of the path lengths from the primary source to the secondary source via intermediate reflecting surfaces and the distance from the secondary source to the receiver.

In the hybrid model described above it is a critical point at which reflection order the transition is made from early to late reflections. Since the early reflections are determined more accurately than the late reflections one might think that better results are obtained with the transition order as high as possible. However, for a given number of rays the chance of missing some images increases with reflection order and with the number of small surfaces in the room. This suggests that the number of rays should be as large as possible, limited only by patience and computer capacity. However, there are two things which make this conclusion wrong. Firstly, the probability of an image being visible from the receiver decreases with the size of the surfaces taking part in its generation, so the number of reflections missed due to insufficient rays will be much fewer than the number of potential images missed. Secondly, in real life, reflections from small surfaces are generally much weaker than calculated by the laws of geometrical acoustics, so any such reflections missed by the model are in reality of less significance than the model itself would suggest. Actually, the efforts of an extended calculation may lead to worse results.

Recent experiments with the ODEON program have shown that only 500 to 1000 rays are sufficient to obtain reliable results in a typical auditorium, and an optimum transition order has been found to be two or three. This means that a hybrid model like this can give much better results than either of the pure basic methods, and with much shorter calculation time. However, these good news are closely related to the introduction of diffusion in the model, as discussed in the following section.

SCATTERING DUE TO ROUGH STRUCTURE OF SURFACES

Diffuse reflections can be simulated in computer models by statistical methods. Using random numbers the direction of a diffuse reflection is calculated with a probability function according to Lambert's cosine-law, while the direction of a specular reflection is calculated according to Snell's law. A scattering coefficient between 0 and 1 is then used as a weighting factor in averaging the coordinates of the two directional vectors which correspond to diffuse or specular reflection, respectively.

By comparison of computer simulations and measured reverberation times in some cases where the absorption coefficient is known, it has been found that the scattering coefficient should normally be set to around 0.1 for large, plane surfaces and to around 0.7 for highly irregular surfaces. Scattering coefficients as low as 0.02 have been found in studies of a reverberation chamber without diffusing elements. The extreme values of 0 and 1 should be avoided in computer simulations. In principle the scattering coefficient varies with the frequency - scattering due to the finite size of a surface is most pronounced at low frequencies, whereas scattering due to irregularities of the surface occurs at high frequencies. However, today's knowledge about which values of the scattering coefficient are realistic is very limited, and so far it seams sufficient to characterize each surface by only one scattering coefficient, valid for all frequencies.

DIFFRACTION DUE TO FINITE SIZE OF SURFACES

The dimensions of a reflector must be seen in relation to the distances a_1 and a_2 to source and receiver, respectively. A useful parameter is the characteristic distance a^* defined by the relatior

$$a^* = \frac{2 a_1 a_2}{a_1 + a_2} \qquad \textbf{(3)}$$

The diffraction from a rectangular panel with two edges perpendicular to the direction from source to receiver can be described by two independent factors, as outlined in ref. [3]:

$$\Delta L_{diffr} = 10 \log (K_1 K_2) \qquad \textbf{(4)}$$

K_1 and K_2 are reflection factors for two infinite strips corresponding to two orthogonal sections through the reflecting surface. Section 1 is supposed to contain the source and receiver points and K_1 is a function of the projection of the panel width $2b\cos\theta$. K_2 is a similar function of the panel length $2l$ in the perpendicular direction.

Approximation for a Long Strip.

A long reflecting strip is a 1-dimensional reflector and $K_2 = 1$ in (4). If the reflection point is located at the centre line of the strip, the reflection can be considered fully effective above a limiting frequency f_{g1}, see ref. [3]:

$$f_{g1} = \frac{c a^*}{2 (2b \cos\theta)^2} \qquad \textbf{(5)}$$

Below this limiting frequency the reflection is attenuated, and a good approximation for the reflection factor is:

$$K_1 \approx \begin{cases} 1 & \text{for} \quad f \geq f_{g1} \\ f / f_{g1} & \text{for} \quad f < f_{g1} \end{cases} \qquad (6)$$

This is valid if the reflection point is at the centre of the strip. If the reflection point is at the edge, the asymptotic attenuation at high frequencies is 6 dB ($K_1 = 1/4$) and the crossover frequency will be two octaves lower than the limiting frequency given by eqn. (5). However, the low frequency behaviour will remain the same.

Approximation for a Rectangular Panel.

This is a 2-dimensional reflector and according to the orthogonality principle two reflection factors must be calculated. K_1 is related to the projection of width $2b\cos\theta$, and is the same as for the long strip above. K_2 is determined from the length of the panel $2l$, and the corresponding limiting frequency

$$f_{g2} = \frac{c\,a^*}{2(2l)^2} \qquad (7)$$

Using the approximate reflection factor (6) for both dimensions will give a combined reflection factor for the panel. The slope of the curve is 6 dB per octave at very low frequencies and 3 dB per octave between the two limiting frequencies. If l and $b\cos\theta$ do not differ too much (say less than a factor of 4) the resulting limiting frequency above which the panel can be considered effectively reflecting, i.e. $K_1 K_2 \approx 1$, is given by:

$$f_g = \sqrt{f_{g1} f_{g2}} = \frac{c\,a^*}{2S\cos\theta} \qquad (8)$$

Although these considerations are related to freely suspended reflectors, it has led to a significant improvement of the ODEON room acoustics model when similar diffraction terms were introduced as a general diffraction attenuation for all surfaces. This means that reflections from small surfaces are attenuated, especially at low frequencies. In this way it can be avoided to find higher order reflections, which are unrealistic because they involve relatively small surfaces, and a more gradual transition from early to late reflections, as described above, has been obtained.

CURVED SURFACES, CONVEX AND CONCAVE SURFACES

A simple geometrical consideration [4] leads to the following approximate term for the reflection from a cylinder with radius of curvature R:

$$\Delta L_{curv} = -10 \log \left| 1 + \frac{a^*}{R\cos\theta} \right| \qquad (9)$$

where a^* is the characteristic distance (3). A convex surface has $R > 0$, whereas a concave surface has $R < 0$. In the latter case energy is concentrated by the reflection, and a focusing effect appears if $R = -a^*/\cos\theta$. In the case of a doubly curved surface with two radii of curvature the attenuation term (9) should be used twice, applying the appropriate projections into the two normal planes of the surface.

The handling of curved surfaces is a major problem in room acoustic computer models. In order to describe the geometry it is normally necessary to subdivide the surface into a number of plane surfaces. But in the case of a convex surface this means, that the reflection is not scattered as evenly as it should be, and in the case of a concave surface it is a problem that each subsurface may contribute a reflection, which means that the total reflection from the curved surface increases with the number of subdivisions made. However, the latter problem is greatly reduced if the diffraction attenuation as described above is introduced in the model.

ANGLE DEPENDENT ABSORPTION

In room acoustic computer models the sound reflections are often treated as mere carriers of sound energy. However, in room acoustics there are many important interference phenomenae, which can only be simulated if the reflections are treated as pressure waves with amplitude and phase. To calculate the sound pressure reflection in a room would need the knowledge of the complex impedances of all surfaces, however, and in general that is not available. The only information that is usually at hand is the statistical absorption coefficient in octave bands, and very often even that is partly a guess. It is not possible to calculate the impedance of a surface from the absorption coefficient, as there is an infinite number of possible combinations of the real and the imaginary part of the impedance which can match the absorption coefficient, even if local reaction is assumed.

A simplified method for modelling the reflection has been proposed [5]. The imaginary part of the impedance is neglected, and the real part is assumed to be independent of the angle of incidence, i.e. the surface is locally reacting. The real part of the impedance is calculated from the measured statistical absorption coefficient by use of a calculated field impedance corresponding to a test surface of about 11 m^2 and an equivalent angle of incidence around 60 degrees. However, it is necessary to decide from the knowledge of the surface material whether the impedance is large or small compared to the characteristic impedance of air, i.e. the material can be either hard or soft.

The general expression for the pressure reflection factor is:

$$r_\theta = \frac{Z_a - Z_f(\theta)}{Z_a + Z_f(\theta)} \qquad \mathbf{(10)}$$

where Z_a is the impedance of the surface and depends on the material, whereas Z_f is the field impedance which depends on the size of the surface and the angle of incidence, θ. For an infinite area $Z_f \rightarrow \rho c/\cos\theta$ and is real. For a finite area, however, the field impedance is complex and finite, but as an approximation it is suggested to consider the real part only, and the following empirical expression has been proposed:

$$Z_f = \rho c \left[\left(\cos^2\theta - 0.60 \frac{2\pi}{ke} \right)^2 + \pi \left(0.60 \frac{2\pi}{ke} \right)^2 + \left(\frac{2\pi}{(ke)^2} \right)^4 \right]^{-\frac{1}{4}} \qquad \mathbf{(11)}$$

Here k is the wave number and e is a characteristic dimension of the surface (the side length of a square with the same area). The characteristic dimension is determined from:

$$e = \frac{4\,S}{U} \qquad \mathbf{(12)}$$

where S is the area and U is the perimeter.

The statistical absorption coefficient α_s under diffuse field conditions is well approximated by an angle of incidence θ = 60°. So, a pressure reflection factor can be estimated by:

$$r_{60} \approx \pm \sqrt{1 - \alpha_s} \tag{13}$$

where r_{60} can be positive or negative. In the present approximation it is assumed to be real, i.e. there is no phase angle but a negative value represents a 180° phase shift of the reflection. From (10) and (13) it is possible to calculate an equivalent real impedance of the reflecting surface:

$$Z_a = Z_f^* \frac{1 + r_{60}}{1 - r_{60}} \tag{14}$$

Here Z_f^* is the equivalent field impedance of the test sample as measured under diffuse field conditions. If the absorption coefficient α_s is assumed to be measured in accordance with ISO 354, the area shall be 10-12 m^2. The proposed values for the equivalent field impedance are given in Table 1.

Table 1. Values of equivalent field impedance corresponding to a test area af 11 m^2.

Frequency, Hz	125	250	500	1000	2000	4000
$Z_f^*/\rho c$	1.04	1.35	1.53	1.62	1.64	1.64

With the approximate method described above it is possible to find an angle-dependent reflection factor, which can typically have a negative sign in case of grazing incidence on a large surface of soft material. However, in order to find the interaction between reflections, some of which have a phase shift, it is necessary to introduce an octave band filtering or similar, for instance by a convolution with the impulse response of an appropriate filter. So, it is possible but relatively complicated to introduce a phase shift in the calculation model. However, there would be several benefits from this, one is the simulation of standing waves in small enclosures and other interference phenomenae, so this would change the low-frequency limit, which is typical for models based on sound energy. This could extend the range of applications of the room acoustical models in the direction of small rooms.

CONCLUSION

The treatment of sound reflections in sound field simulations has been outlined. Most of the problems are related to the fact that sound is a wave phenomenon, whereas the generally applied simulation technique is based on the assumption of some kind of ray or particle phenomenon. The proper treatment of these problems may lead to better and more reliable models.

REFERENCES

1. J.H. Rindel, "Diffusion of Sound in Rooms - An Overview". 15th ICA, Trondheim, Proceedings **2**, 633-636 (1995).
2. J.H. Rindel, "Computer Simulation Techniques for Acoustical Design of Rooms". Acoustics Australia **23**, 81-86 (1995).

3. J.H. Rindel, "Acoustic Design of Reflectors in Auditoria". Proceedings, Institute of Acoustics **14,** 119-128 (1992).
4. J.H. Rindel, "Attenuation of Sound Reflections from Curved Surfaces", Proceedings of 24th Conference on Acoustics, Strbské Pleso, 194-197 (1985).
5. J.H. Rindel, "Modelling the Angle-Dependent Pressure Reflection Factor". Applied Acoustics **38**, 223-234 (1993).